Quantifying the complexity of 3D printed concrete elements

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Abstract -

Freedom of shape enabled by 3D concrete printing (3Dcp) is often mentioned alongside productivity, technology progress, material optimization, and other benefits of the technology. When doing so, printed structures are described using qualitative terms such as "complex", "double-curved," and "geometric freedom". However, such descriptions depend on the aesthetics and the observers' interpretation, which renders an objective comparison between concrete objects difficult. To alleviate the ambiguities with such qualitative comparisons, this study proposes a quantitative metric consisting of two complexity coefficients – Intrinsic Complexity Coefficient (ICC) and Fabrication Specific Complexity Coefficient (FSCC). The ICC considers the concrete elements' geometry using shape coefficient and mean curvature, whilst the FSCC defines the elements' complexity within the context of 3Dcp (ease of printing, resolution). Thus, the ICC can be used to compare printed elements and the FSCC to determine which element is easier to print. Within this study, 10 pillars with varying complexity were designed and then graded according to the two complexity coefficients. Further, this evaluation was employed to adjust the construction duration consumed when calculating the productivity of elements produced using 3Dcp and traditional construction techniques. In such a way, the coefficients allowed to incorporate geometric complexity when comparing the productivity of various construction techniques, illustrating just one of many applications for ICC and FSCC.

Keywords -

3D Concrete Printing, Digital Fabrication, Freedom of Shape, Quantification of Geometric Complexity

1 Introduction

One of the most significant advantages of 3Dcp is its ability to create concrete elements of great geometric freedom. This is particularly relevant as traditional construction techniques fail to do so because of difficulties in manufacturing complex formwork, which is conventionally made from plywood, steel, or aluminum. On top of that, formwork sizes are standardized, which further limits the variety of shapes that can be manufactured. This is done to maximize their reuse, thus, minimizing the formwork cost per element. Nevertheless, even with simple shapes, formwork costs are significant and may account for 35-54% of the total construction cost and consume 50-75% of the total construction time [1].

Further, concrete is a convenient material to use with various formwork shapes due to its ability to flow into place before curing. However, this free-flowing ability is often not used to its full potential with conventional rectilinear and cylindrical formwork. This opportunity seems wasted even more since 3D parametric modeling tools provide architects and structural engineers with enhanced design flexibility.

At this moment, concrete elements of conventional shapes can still be produced more productively using in-situ or precast methods when only the construction phases are considered [2]. More comprehensive studies have shown that alternative methods, such as the Mesh Mould [3], become more feasible when the complexity increases [4]. This leads to believe that rather than perceiving 3Dcp as a replacement for conventional construction techniques in normal ambient conditions, it should currently be perceived as an alternative for specific jobs that, for instance, require the ability to build geometrically more complex shapes. However, due to difficulties in quantifying the complexity of such shapes, printed elements that differ from conventional forms, such as a straight wall, are described only using qualitative terms like "complex", "double-curved" and "geometric freedom". Such descriptions are greatly dependent on the aesthetics and the observers' interpretation of the complexity of a given element. Therefore, any analysis of how complex the respective elements are, is highly subjective. On top of that, it is difficult to provide a fair comparison between 3Dcp and traditional methods of production as it is not always possible to capture the added benefit of printing complex geometries. Thus, this study proposes two coefficients aimed to quantify complexity and allow the above-mentioned comparisons.

Of course, 3Dcp offers other benefits asides from creating more geometrically complex elements. Nevertheless, exploring these benefits is outside of the scope of this paper.

2 Methodology

This study proposes two coefficients for evaluating the geometric shape complexity of a concrete element. First, the Intrinsic Complexity Coefficient (ICC) captures the geometric complexity of the element itself while the Fabrication Specific Complexity Coefficient (FSCC) relates to how difficult the element is to print. This section describes how the two coefficients are derived and how they can be applied to a productivity analysis using an example of 10 pillars designed within this study.

2.1 Designing 10 Pillars with Varying Geometric Complexity

10 pillars were designed as part of this study using Rhino 6, a 3D computer aided design software, and its in-built plug-in Grasshopper, a visual programming environment [5]. These pillars, shown in Figure 1, display varying levels of geometric complexity that can be created using 3Dcp. All of the pillar designs stem from the same parametric Grasshopper script [6], in which multiple parameters such as the degree of the NURBS curves, radius of cross sections at different heights, and number of side ornaments were varied to create 10 different pillars. The pillars are not designed to carry any axial, shear, bending or torsional loads and are intended for architectural purposes only. Design was limited so that that overhangs could not be printed at angles greater than 10.6° due to printing and material constrains. All pillars are 200 cm tall, with a varying diameter from 25 to 44 cm.

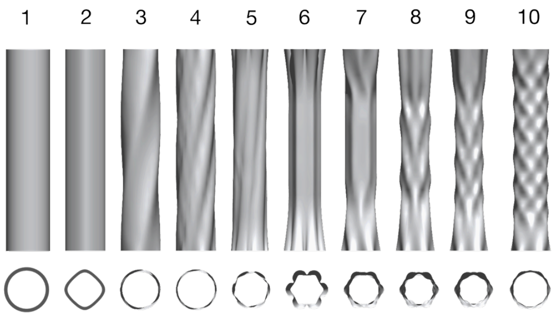

Figure 1. 10 pillars of varying geometric complexity

2.2 Intrinsic Complexity Coefficient

To counter the ambiguity that stems from describing concrete shapes using qualitative terms, we introduce a quantitative geometry-specific Intrinsic Complexity Coefficient (ICC). We suggest that the ICC is dependent on the local geometries that make up the elements. More specifically, we classify the local geometries using the Shape Index (SI) and the mean curvature of the local geometries. The reasons for choosing these parameters and how they are combined are described in the following sections. The conceptual make-up of the coefficient can be seen in Figure 2.

By employing the ICC, a more systematic method of comparing different concrete elements can be introduced. This is relevant, for example, for comparing the levels of development of 3Dcp technology over time, which can be done by noting the range in ICC of elements that the technology is able to manufacture.

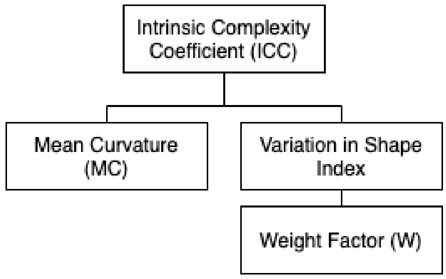

Figure 2. Components of the Intrinsic Complexity Coefficient (ICC)

2.2.1 ICC: Shape Index, SI

The Shape Index (SI) is a non-dimensional and scale-invariant coefficient in the range [-1, 1] [7]. The lower end of the range (-1) represents a spherical cup (concave) while the higher (1) a spherical cap (convex). The shapes with an SI in between the two limit values vary, representing shapes such as a rut, saddle, ridge, dome etc. The index describes the shape of a local geometry using the principal curvatures of the locality (Equation 1) [7]. Instead of using several values to describe a local geometry, SI provides a single encompassing index that reflects the shape of the local geometry independent of its size.

$$SI = \frac{2}{\pi} \arctan\left(\frac{k_2 + k_1}{k_2 - k_1}\right) \tag{1}$$

where $k_1 \geqslant k_2$ and k_1 and k_2 describe principal curvatures.

For the ICC, we calculated the SI of each pillar at equally spaced points on the pillars' surfaces. In this study, we used 5050 points as the computing power did not allow for more. However, in the future, a specific number of points per surface area should be chosen for more consistent comparisons. The SI was used to map the variation in local geometries for each pillar by obtaining ranges of the SI. The ranges with at least 1% of the total points were used for further calculations thus discarding local geometries that appeared in insignificant amounts. For each range the mean curvature was calculated and the percentage of points in a certain range was used to obtain the weight factor. This is explained in detail in section 2.2.3.

2.2.2 ICC: Mean Curvature, MC

To understand why curvature can serve as a relevant metric in determining the complexity of an element, we first define what constitutes a simple shape. In this regard, straight lines and flat planes intuitively seem like the most simplistic shapes in 2D and 3D spaces, respectively. More complex shapes are achieved from altering a line or a plane, which can be observed in the increase in difficulty in the mathematical expressions of such elements. Based on this, a parameter like the curvature, which describes the extent of which a shape differs from a flat object (i.e., line or plane), can be used to describe the complexity of a shape. Two types of curvatures are defined in differential calculus: intrinsic and extrinsic [8]. Intrinsic curvature relies solely on the algebraic properties of the surface itself and does not require any knowledge of how it is embedded in the surrounding space. A type of this curvature is the Gaussian Curvature. In simple terms, it describes whether an object could be transformed into a flat plane without stretching the surface of the respective object. If the Gaussian Curvature is zero, one of the principal curvatures must have been 0 and the surface can be rolled out into a flat plane. To demonstrate, we can imagine a sheet of paper rolled to form a cylinder. Although the cylinder is curved, the paper can easily be laid flat, thus, the Gaussian Curvature equals zero. Within the context of this study, we would consider a cylinder to have a higher curvature than a plane, thus, the extrinsic curvature is more appropriate than intrinsic when considering the local geometries.

The extrinsic curvature can be described using the mean curvature, which is the average of the two principal curvatures (Equation 2). It describes the curvature of an element based on its surface properties and the fashion in which it is embedded in space.

$$MC = \frac{k_1 + k_2}{2} \tag{2}$$

where k_1 and k_2 are the principle curvatures.

2.2.3 ICC: Weighing and forming the coefficient

As some of the local geometries are more ubiquitous in the concrete element than others, we introduce a weight factor, W, that is dependent on the number of respective local geometries in the element ($n_{\ local\ geometry\ l}$). It is defined as the natural logarithm of the percentage of shapes that are within the respective SI range squared (Equation 4). To not skew the results by local geometries that appear only a small fraction of times, only the ranges with at least 1% of the total measured local geometry count are included. For a similar reason, the natural logarithm is squared to make sure that the coefficient scales over a greater range and gives greater weight to local geometries that are more present within the element.

$$N_l = \frac{n_{\ local\ geometry\ l}}{n_{\ all\ local\ geometry}} * 100\% \tag{3}$$

where $N_l \geqslant 1$. N_l is the percentage of local geometries representing a specific local geometry l and n represents number of local geometries.

$$W_l = ln(N_l)^2 \tag{4}$$

where W_l represents the weight of the local geometry shape l.

In addition, the average mean curvature for every SI range is calculated by summing all the mean curvature values for every local geometry within a range and then dividing by the number of local geometries within the same range (Equation 5).

$$MC_l = \frac{\sum_{i=1}^{N_l} MC_i}{N_l} \tag{5}$$

Once the average mean curvature value for points in an SI range is obtained, we multiply the value with the weight for the respective range. These multiplication values for all SI ranges are then added together to find the ICC. The code for obtaining the values for calculating the ICC can be located in [6].

$$ICC = \sum_{l=1}^{L} (MC_l * W_l) \tag{6}$$

2.3 Fabrication Specific Complexity Coefficient

While the ICC can assist in comparing produced elements, it cannot be used to determine which design is easier to fabricate using 3Dcp. Therefore, we suggest a Fabrication Specific Complexity Coefficient (FSCC) to measure how complex the printing process of a concrete element is.

To better illustrate the need for such a coefficient, we can imagine a cone-like concrete element. The ICC would be the same regardless of whether the cone is placed standing on its larger base or on its smaller base. However, if the cone had to be printed standing on its smaller base, gradually increasing in size as the element is printed, it would be more difficult to fabricate than if the cone was placed on its larger base. This demonstrates the need for FSCC. Using both coefficients in junction can be useful when comparing existing 3Dcp technologies - their capability of creating an object of a certain ICC by using effort described by FSCC.

FSCC is suggested to be a function of how easy the element can be printed (i.e., ease of printing) and how well the final element will reflect the parametric model (i.e., resolution). The breakdown of the FSCC is shown in Figure 3.

These properties were obtained by performing simulations in Grasshopper. They are described in greater detail in the following sections.

Table 1. Properties used in the simulation to quantify the ease of printing, where t is curing time in minutes[9]

	Property	Quantity
Element Geometry	Object Height	$2m$
	Object Diameter	$25-44cm$
Printer Properties	Layer Height	$1cm$
	Printer Speed	$0.5mm/s$
Material Properties	Specific Weight	$20.2kN/m^3$
	Young's Modulus	$0.0781+0.0012*t$ MPa
	Shear Modulus	$\frac{0.0781+0.0012*t}{2.6}$ MPa
	Yield Strength	$5.984+0.147*t$ KPa
	Coef. of Thermal Expansion	$0.00001°C^{-1}$

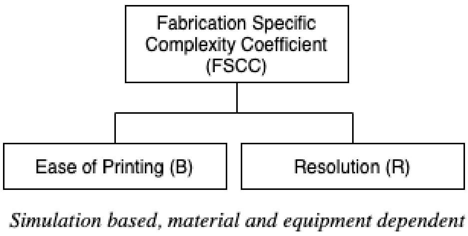

Figure 3. Fabrication-Specific Complexity Coefficient, FSCC

2.3.1 Ease of printing, B

The ease of printing is dependent on a multitude of variables such as the:

- material properties (yield strength, Young's modulus, stiffness and the rate of change of these properties over time while the material solidifies),
- printer properties (rate of extrusion and printed layer dimensions),
- geometry of the element (size and curvature).

To account for all of them, we used a simulation script in Grasshopper created by Witteveen+Bos, Karamba3D, Nanyang Technological University, and used in studies at the Eindhoven University of Technology (TU/e) [9]. It analyzes when a printed element would buckle during the printing process and accounts for the variables mentioned above. In this study we focused on the height of the element that can be printed before it buckles as the output of this simulation. The original authors of the simulation empirically found that when using their materials and technologies an element would buckle when the displacement exceeds 1 cm. For simplicity, all the properties used in the simulations in this study are the same as originally assigned by the authors of the simulations. The only exception is the printing speed, which was decreased to exemplify the difference in the height that can be printed before an element buckles. These parameters are shown in Table 1.

These properties can be adjusted based on the materials and technologies used. Similarly, the 1 cm limit for buckling varies based on different properties. Because of this variability, the same concrete element can have a different ease of printing and, thus, a different FSCC if the elements are manufactured at different facilities, unlike the ICC, which remains the same.

2.3.2 Resolution, R

Similar to how a picture's resolution determines how well details are visible in a photo, the dimensions of the printed layers determine how well the final printed element will resemble the parametric model. It is important to account for this, as the height of the printed concrete layers can restrict how geometrically intricate a printed concrete element can be. Thus, we suggest considering the volumes of the parts of the printed element which differ from the parametric model. This idea is demonstrated in Figure 4, which shows pillar 10 if it would be printed with a layer height of 4 cm. The green color indicates the excess printed concrete and red color shows the concrete missed in relation to the parametric model.

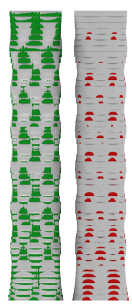

Figure 4. Extra concrete printed (green) and concrete missed (red) for pillar 10 if it would be printed with a layer height of 4 cm

To achieve a measure of resolution, we created a Grasshopper script [6] which identifies these parts (of extra concrete and omitted concrete) and adds their volumes. In this script, it is assumed that no sagging of concrete occurs. The percentage difference of concrete volume in the parametric model and the printed element is used as the final value.

2.3.3 FSCC: Combination of the parameters

Both the ease of printing (B) and the resolution (R) are expressed as percentages before they are used to calculate the FSCC. In the former case, the height at failure is used to calculate the height left until the top of the element. This difference is then used to measure what percentage of the pillar was left until completion as shown in Equation 7.

$$B = \frac{h_{total} - h_{failure}}{h_{total}} * 100\% \tag{7}$$

where h_{total} is the height of the pillar and $h_{failure}$ is the height at which the pillar will collapse (obtained from the simulation [9]).

Similarly, to account for the resolution, the volume of the concrete that is printed and/or omitted with respect to the parametric model is expressed as a percentage fraction of the total concrete pillar volume (Equation 8).

$$R = \frac{V_{extra} + V_{omitted}}{V_{total}} * 100\% \tag{8}$$

In both cases, the larger the value, the greater the complexity of the element that the parameter is describing. Finally, the FSCC is obtained by multiplying both results as shown in Equation 9.

$$FSCC = B * R \tag{9}$$

A larger FSCC indicates greater complexity in 3D printing an element.

2.4 Productivity Analysis

An example of the use of the ICC is within productivity analysis. Productivity data can be adjusted using the ICC to provide a fairer comparison between different production techniques. When producing more complex elements (or elements with a higher ICC), the process is more productive as, in this time, a more complex design is achieved than that of a simpler one (e.g., circular column). Thus, a scaling factor is introduced to reduce the time to account for the added complexity in the productivity comparison. The scaling factor, f, is obtained as shown in Equation 10.

$$f = ICC + 1 \tag{10}$$

The adjusted time is not meant to be a reflection of real-life production duration, but a measure used for comparing productivity. It is calculated as shown in Equation 11.

$$T_{adj} = \frac{T}{f} \tag{11}$$

where T_{adj} is the adjusted time, T is the real-life duration of production and f is the scaling factor.

3 Results

Pillars of varying geometric complexity were designed with Rhino 6 and Grasshopper [5]. The following sections show the complexity analysis of these pillars as outlined in the Methodology, as well as the implementation of ICC in productivity analysis.

3.1 ICC – Results

The shape index was obtained by following the process outlined in Section 2. Although variance of SI was not directly used in obtaining the ICC, the weight factor is influenced by how varied local geometries are. The ICC will be increased in value when there is a larger variety in the shapes of local geometries in the object. This is confirmed in Figure 5 where the relationship between variance of the shape index and pillar number is illustrated. A general trend can be noticed, where, with the exception of pillars 5 and 6, the variance of the shape index (SI) increases with the pillar number, similar to how the ICC increases with the pillar number (Figure 7).

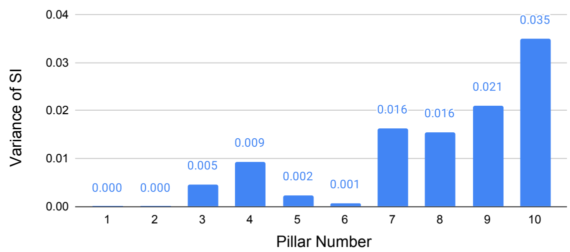

Figure 5. Variance of Shape Index for Pillars 1-10

The mean curvature was obtained in Grasshopper and is shown for each pillar in Figure 6.

The results for the shape index as well as the mean curvature were weighed and combined to give the values of the Intrinsic Complexity Coefficient (Figure 7). It can be seen that although shape index variance was very low for pillars 5 and 6, the mean curvature is the highest for these pillars, resulting in comparable values for their ICC. This shows the trade off between variation in local geometries and mean curvature that we had during the design of the 10 pillars. This is because printing an object with many

Table 2. Pillar parameters required for the FSCC

Pillar #	Volume difference (%) (Layer h = 1 cm), R	Height at failure (cm)	Percentage of total height not reached (%), B	FSCC	Rank
1	0.10	192	0.04	0.00	1
2	0.10	168	0.16	0.02	2
3	0.18	168	0.16	0.03	3
4	0.60	172	0.14	0.08	4
5	0.45	156	0.22	0.10	5
6	0.84	152	0.24	0.20	7
7	0.79	144	0.28	0.22	8
8	1.07	144	0.28	0.30	10
9	0.55	140	0.30	0.17	6
10	1.27	156	0.22	0.28	9

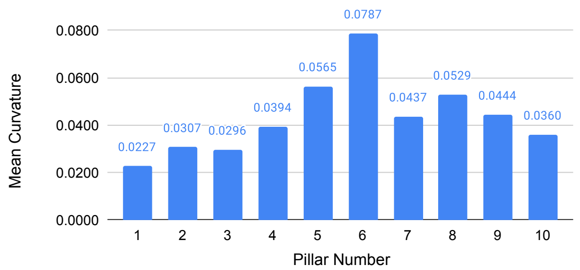

Figure 6. Mean Curvature for Pillars 1-10

different local geometries that would also have large mean curvatures was not a viable design option due to the restrictions on maximum printing angle for the 3D printer at the Besix3D facility [10].

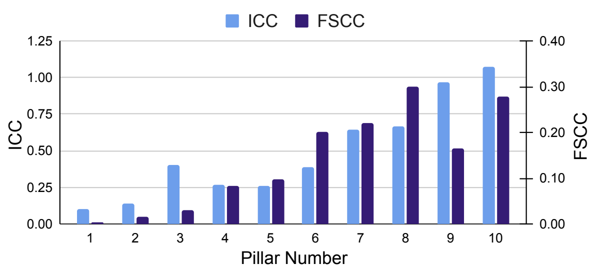

Figure 7. ICC and FSCC of each pillar

Pillars 9 and 10 obtained the highest values for the ICC, as they incorporated both varied local geometries and large mean curvature. Pillars 1 and 2 had the lowest ICC values as expected due to their simple design (circular pillar and rectangular pillar with rounded corners).

3.2 FSCC – Results

The parameters for each pillar required for obtaining the FSCC are listed in Table 2. By multiplying the percent volume difference with the percentage of total height left to print, we can obtain the FSCC. Similarly, we display these coefficients in Figure 7 to visualize the differences in the fabrication complexity between the different pillars.

As expected, the FSCC for the first two pillars is the lowest. These pillars could also be constructed using in-situ and precast construction techniques with no foreseeable difficulties. Pillar 8, is the most difficult to construct as per the FSCC.

The results show that the ICC and FSCC have an R^2 value of 0.60, meaning that 60% of the FSCC data can be explained by the ICC.

3.3 Productivity Analysis

Data from a productivity study that compared the productivity of constructing a column with 3Dcp, precast and in-situ techniques [2] was employed. The data was adjusted using the scaling factor obtained from the ICC. By implementing this scaling factor, the increased complexity - the increased ICC value - for the 3Dcp columns was taken into account, acknowledging the added benefit of the design for the 3Dcp columns, thus, providing a fairer comparison between the three production techniques. Although the elements in this study are of a different design than in the productivity study, the production time for 3Dcp is more dependent on the printing speed and volume of material used. As the dimensions are comparable between the 10 columns designed as part of this study and the Concrete Choreography columns used in the productivity study [2] [11], the duration can be assumed to be the same for this analysis. Results can be seen in Table 3.

The time (T) is taken from the productivity study [2], the ICC for the in-situ and precast columns is assumed to be 0.105, which is the same as the ICC for pillar number 1, as all of these are circular columns of relatively similar size. The factor f and $T_{adjusted}$ are calculated as outlined in the previous section.

It can be seen that after scaling, 3Dcp becomes the more productive production method for some of the columns.

Table 3. Scaled time of production

Method	T(h)	ICC	f	$T_{adjusted}$(h)
In-situ	54.2	0.105	1.11	49.05
Precast	50.7	0.105	1.11	45.88
3Dcp	59.0	0.105	1.11	53.39
		0.140	1.14	51.75
		0.258	1.26	46.90
		0.267	1.27	46.57
		0.385	1.39	42.60
		0.404	1.40	42.02
		0.645	1.65	35.87
		0.668	1.67	35.37
		0.965	1.97	30.03
		1.071	2.07	28.49

Productivity of in-situ is exceeded for pillars with ICC 0.258 and higher and productivity of precast is exceeded for pillars with ICC 0.385 and higher.

4 Limitations

There are several limitations to this study. First, the coefficients were created empirically by considering the factors that influence complexity and describe a 3D shape. All of the decisions made in the creation of the coefficients had logical reasoning as well as were generally backed by mathematical theory. However, this approach could have resulted in a biased and partial selection or omission of elements that make up the ICC and FSCC, as well as the way the coefficients are weighed.

Second, the ICC is made to evaluate curved designs. If a design would include sharp edges and straight planes, the ICC would be significantly lower. This is because in this study we define the opposite of complexity to be a straight line or plane. Thus, for future studies, another element in the ICC should be introduced to consider a design with straight edges to also be of a certain complexity.

Third, only 10 elements are evaluated raising concerns of a lack of statistical relevance. Although the obtained results follow expectations and work well with the designs in this study, the coefficients should be tested on more elements of different dimensions and designs to verify their functionality or find ways to improve them.

Last, in the productivity analysis several values were approximated or assumed based on similarity, such as the ICC values of the in-situ and precast columns or the printing time for the 3Dcp pillars. Nevertheless, these are reasonable and the printing time for the 10 pillars is more likely overestimated than underestimated due to the Concrete Choreography columns [2][11] being larger. Thus, in reality the 3Dcp pillars' values for $T_{adjusted}$ would likely be lower than what they are in this study, resulting in 3Dcp still proving to be more productive than traditional methods for pillars with higher ICC.

5 Discussion

This study offers two coefficients - ICC and FSCC - the former to quantify the complexity of concrete elements and the latter to quantify the effort in producing them using 3Dcp. Within the scope of this study, the coefficients proved effective, providing results that aligned with expectations and could be employed for productivity analysis.

The relationship between the ICC and FSCC was explored. For the 10 pillars used in this study, it was found that ICC and FSCC had an R^2 value of 0.60. The coefficients are not strongly correlated, yet show a generally similar trend where 60% of the variation in the ICC can be explained by the FSCC values. This supports the idea that for 3D printing concrete elements, the shape/design of the element (as long as it fits within the constraints of the printer, such as overhangs and maximum angles) does not directly impact the difficulty of printing. This contrasts with traditional methods, where generally, an increase in shape complexity results in increased difficulty in production, leading to additional costs.

The FSCC results also align with our expectations, as they generally increase with the ICC of the pillar. It is important to note that the coefficient does not only depend on the design but also the materials used and the printer's parameters, such as speed of extrusion, as they are significant factors in the buckling simulation. Thus, FSCC can be considered a good representation of the difficulty of the exact manufacturing process planned - specific to the facility, technology, and materials chosen. This enables to use FSCC for choosing the printing process for a specific object, including the choice of material and printing settings. Furthermore, FSCC can be used to compare 3D printing technologies and their development by looking at how FSCC would decrease for the same concrete element when printed with improved materials/printers. In future studies, FSCC could be created for various techniques (e.g., in-situ, precast) to allow direct comparison and aid in choosing a production method.

The coefficients were only tried on the 10 designed pillars, but they can also be applied to other construction elements. The transferability was not explored in this study, but we expect no significant difference of the ICC and FSCC's applicability for different elements. The only issue that may arise is that the coefficients are not normalised, thus, elements might be difficult to compare if the objects are of very different dimensions.

Lastly, productivity analysis was carried out for the 10 pillars as well as columns constructed with in-situ and precast methods. The results showed that, after scaling, 3Dcp does, in fact, become more productive than in-situ and precast methods for pillars with higher ICC values. This aligns well with expectations, as pillars with the two lower ICC values could be produced using traditional construction techniques and would likely be less costly and time-consuming than if produced with 3Dcp. However, the pillars with higher ICC values would be extremely difficult, if not impossible, to produce with the traditional methods. Thus, the analysis supports the notion that for more complex designs (higher ICC) 3Dcp is more productive than traditional construction methods. It needs to be taken into account that many values used in this analysis were approximated. To obtain the exact values, the pillars would need to be printed, and the process for calculating the ICC would have to be carried out for the circular column of the exact dimensions as used in the productivity study [2]. The example was shown as a preliminary attempt at considering complexity within productivity, with the hope to aid in the implementation of such creative and original designs in construction projects.

6 Conclusions and Outlook

Overall, the two complexity coefficients - ICC and FSCC - are a good start in quantifying the complexity of concrete elements, yet require further testing and perhaps alterations. With parametric design as well as different manufacturing methods being developed and implemented in industry, it is important to perform not only a qualitative but also a quantitative comparison between designs and production methods. An example of how ICC could be employed in productivity analysis was shown, providing results that aligned with expectations. The adjusted time showed that after scaling, for pillars with higher ICC values, 3Dcp is the more productive method of construction when compared to precast and in-situ.

In future studies, the coefficients could be improved by changing the way the weights of the variables are distributed, as well as by altering the variables that were included in the coefficients. Nevertheless, for the scope of this study, the ICC and FSCC can be considered an appropriate representation of the complexity of a concrete element and the ease of manufacturing it using 3Dcp.

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